The Big Picture

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One way to look at the end goal of studying GR is that we want to be able to "do physics" in four dimensional curved spacetime. Let's look at mechanics as an example. In non-relativistic physics we can mostly use Newton's 2nd law:

$$\frac{d^2x}{dt^2} = \frac{F}{m} \quad \text{or} \quad \frac{d^2x}{dt^2} = -\frac{1}{m}\nabla P$$

where x and F are vectors and P is the potential energy function.

The corresponding equation of motion in GR is the *geodesic equation*

$$\frac{d^2x^\mu}{d\lambda^2} = -\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}$$

This tells us the acceleration of an object (due to gravity) in spacetime. In GR gravity is not considered a force. Instead we say that an object (which is not being subjected to any forces) travels on the shortest path between two points. This path is given by the geodesic equation.

Notice how the GE is similar in form to the "gradient" version of Newton's 2nd law. x still represents the position of the object (but it's now a point in 4D spacetime). λ is not exactly time, but it is still the independent paramater which lets us draw the parametric curve of a path. In many cases it will be proper time, but not always.

Γ is a rank-three tensor, which can be derived from the metric tensor g.

So the upshot is that once we find out what g is then we can construct the equation of motion in curved spacetime.

We find out what g is by solving Einstein's equation:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

I won't try to unpack this equation now, but essentially g depends on the distribution of mass/energy/momentum throughout spacetime.

Starting with the metric tensor and then calculating motion is the easy part. Starting with mass/energy and solving Einstein's equation to *find* g is the hard part.