# June 23rd Notes

June 28, 2019

## The Geodesic Equation

*A geodesic is a path which parallel transports its own tangent vector.*

So if no force it applied, then the only acceleration is that which is inherent in the geometry of spacetime. This is the variation from flat spacetime which is captured by the Christoffel symbol.

$$\frac{d^2x^\mu}{d\lambda^2} = -\Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}$$

## Calculation of the Christoffel symbol

$$\Gamma^\mu{}_{\alpha\beta} = \frac{1}{2}g^{\mu\rho}\left(\frac{\partial g_{\alpha\rho}}{\partial x^\beta} + \frac{\partial g_{\beta\rho}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\rho}\right)$$

## The Covariant Devivative

$$\nabla_\alpha V^\mu = \frac{\partial V^\mu}{\partial x^\alpha} + \Gamma^\mu{}_{\alpha\beta}V^\beta$$

## Other facts about the Christoffel or Connection symbol

$$\Gamma^\mu{}_{\alpha\beta} = \frac{\partial x^\mu}{\partial V^\rho}\frac{\partial^2 V^\rho}{\partial x^\alpha \partial x^\beta} \quad - \text{Neuenschwander page 100 (4.15 )}$$

$$\frac{\partial e_\alpha}{\partial x^\beta} = \Gamma^\mu{}_{\alpha\beta}e_\mu \quad - \text{Collier page 148 (6.2.2)}$$