Non-locality II: GHZ Measurements

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Background

In this series we're interested in the non-locality question: Whether what we know about quantum physics implies that there is some kind of nonlocal aspect to the world. The mathematics of quantum theory predicts correlations for entangled objects which are surprising, since the result of a measurement on one object *appears* to depend on what type of measurement is done on the other one. The two most obvious explanations for these correlations are (1) the objects communicate with each other, so that each party knows what type of measurement the other is doing, or (2) the states of the objects are determined in advance in a way that predetermines the correct values for all possible measurements.

The standard reading of relativistic physics *seems* to tell us that communication is off the table. Not so much because of the speed with which it might happen, but rather that the inability to specify which measurement happened first makes a cause and effect relationship appear incoherent.

The term *hidden variables* refers to the second option. These "hidden" variables are simply the predetermined values which will be observed when a specific measurement is done. What John Bell is primarily famous for is giving a rigorous demonstration that, for certain quantum experiments, given certain reasonable sounding assumptions, no such combination of predetermined values can possibly account for such results.

Bell's theorem, however, depends on a non-trivial statistical argument. Since that time, a number of simpler demonstrations have been discovered which serve the same purpose. The GHZ experiment is one of these. What Greenberger, Horne, and Zeilinger discovered is that it's possible to rule out predetermined values with a single experiment. No statistical argument is required.

The GHZ State

The form of the GHZ state that we're going to use is simply an equal superposition of three zero bits and three one bits:

$$|ghz\rangle = \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle$$

If we write it as Z spin it looks like this:

$$|zzz\rangle = \frac{1}{\sqrt{2}}|+z,+z,+z\rangle + \frac{1}{\sqrt{2}}|-z,-z,-z\rangle$$

So far, nothing special.

The Experiment

Let's say that the particles belong to Alice, Bob, and Carol. They get together and entangle them in the GHZ state. Then they proceed to distant locations and one of the four following combinations of measurements is done:

(1) Alice, Bob, and Carol all do a spin X measurement (XXX).
(2) Alice measures spin X while Bob and Carol measure spin Y (XYY).
(3) Alice does Y, Bob does X, and Carole does Y (YXY).
(4) Alice and Bob both do Y and Carole does X (YYX).

In order to see what the possible results of the different measurement combinations are, we need to change the basis of the GHZ state in four different ways:

$$|xxx\rangle = \frac{1}{2}|+x,+x,+x\rangle + \frac{1}{2}|+x,-x,-x\rangle + \frac{1}{2}|-x,+x,-x\rangle + \frac{1}{2}|-x,-x,+x\rangle$$

$$|xyy\rangle = \frac{1}{2}|+x,+y,-y\rangle + \frac{1}{2}|+x,-y,+y\rangle + \frac{1}{2}|-x,+y,+y\rangle + \frac{1}{2}|-x,-y,-y\rangle$$

$$|yxy\rangle = \frac{1}{2}|+y,+x,-y\rangle + \frac{1}{2}|+y,-x,+y\rangle + \frac{1}{2}|-y,+x,+y\rangle + \frac{1}{2}|-y,-x,-y\rangle$$

$$|yyx\rangle = \frac{1}{2}|+y,+y,-x\rangle + \frac{1}{2}|+y,-y,+x\rangle + \frac{1}{2}|-y,+y,+x\rangle + \frac{1}{2}|-y,-y,-x\rangle$$

$|xxx\rangle$ is a standard change of basis from Z to X, but the other three are portrayed in a *mixed basis.* (This has nothing to do with a mixed state, these all represent the same pure state!) There are a couple of techniques for doing the "mixed" change of basis. I'll show them on the board if anyone is interested. Any mechanism for doing the basis change by hand is quite tedious.

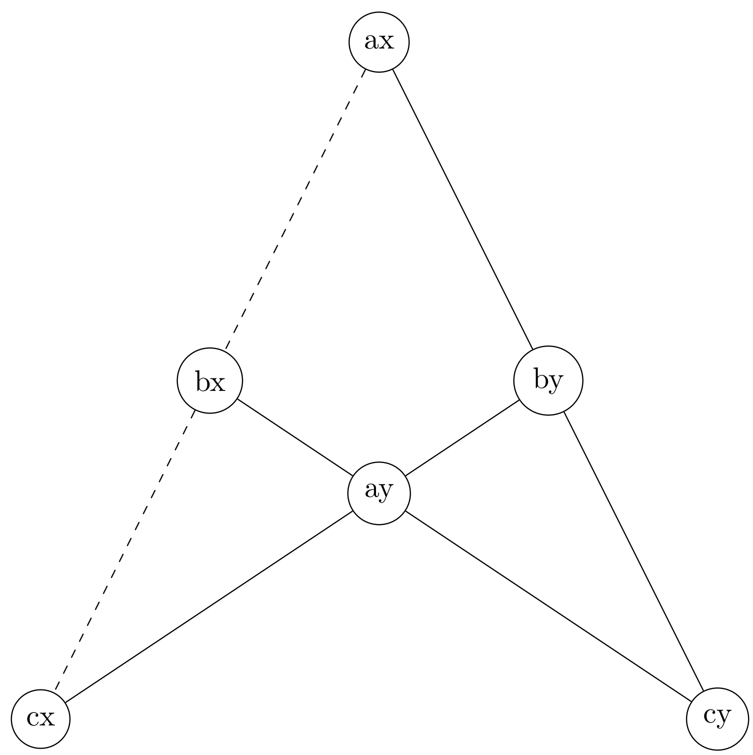
Now the question before us is whether Alice, Bob, and Carol can set up their systems (bits or particles) in advance in such a way that whichever of the four sets of measurement is done, a possible state will result. (Possible in the sense that it's one of the parts of the superposition for the measurements in question.) For example, if the YXY measurements are done, the results can only be $|++-\rangle, |+-+\rangle, |-++\rangle$, or $|---\rangle$.

We can assume that each of the participants has two variables, one tells what to return for an X measurement and the other tells what to return for a Y measurement. So we have six (binary) variables. Let's call them: ax ay bx by cx cy. All in all, there are 64 different ways the three participants can set things up in advance. You could go through all 64 of them and try each one against each of the four states. But there's a way to demonstrate that it can't be done, based on the patterns of + and - in the states.

Notice that there are really only two patterns of results:
For $|xxx\rangle$ we have: $|+++\rangle, |+--\rangle, |-+-\rangle$, or $|--+\rangle$.
For all the others: $|++-\rangle, |+-+\rangle, |-++\rangle$, or $|---\rangle$.

Key point: the $|xxx\rangle$ measurment must have an odd number of pluses, and all the others an even number of pluses.

We can depict all the pertinent information on the following diagram. The circles represent all of the variable settings and the lines represent the measurements. The dotted line is $|xxx\rangle$ and the three solid lines are $|xyy\rangle, |yxy\rangle$, and $|yyx\rangle$.



The solid line measurements all get an even number of + results, and the dotted line an odd number. So if there is any way that Alice, Bob, and Carol can fill in their six variables successfully, then when you add up the number of + results in all four lines, the total (three even numbers plus one odd number) needs to be odd.

But this is impossible, because (as you can check) when you add up all four lines, each circle is counted exactly twice. So no matter how things are arranged, adding up all four will get an even number of pluses.

This shows that there is no way the variables (and hence the results of the measurements) can be set in advance, in such a way that they will achieve the same results that quantum theory predicts.

Sources

The diagram comes from Tim Maudlin's *Philosopy of Physics: Quantum Theory* where the GHZ discussion starts of page 29 of the copy I have. Maudlin credits David Mermin with coming up with the basic even/odd argument. Other references that talk about GHZ are Maudlin's *Quantum Non-Locality & Relativity* and David Mermin's *Quantum Computer Science*.