Non-locality I: Introduction

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The concept of hidden variables

Let's say someone flips a coin and covers it up as soon as it lands. We know that it's either heads or tails, but we don't know which. Knowing how coins work, if we had to bet one way or the other we'd give 50/50 odds. In a probabilistic sense, we could say that the "state" of the coin is:

$$\text{Coin} = 50\%\ \text{Heads}\ ,\ 50\%\ \text{Tails}$$

But really it is one or the other.

Similarly, when a qubit is in an equal superposition of zero and one:

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

We know that when we measure it we have a 50/50 chance of seeing zero or one.

Since the beginning of quantum physics, there has been an ongoing debate about whether the quantum case (superposition) is in some sense equivalent to the classical case (probability of a specific, but unknown outcome).

Some take the position that quantum theory is *complete* and that the superposition above describes everything. The bit simply isn't really zero or one until we measure it.

Others take the position that quantum theory is *incomplete.* The bit really is either zero or one, but the theory is missing something that would tell us which.

The second position, that the theory is incomplete, sometimes goes under the name *hidden variables.* In this case, the variable that's "hidden" is the actual binary state of the bit, either one or zero. It's not hidden from *us.* We see it when we do the measurement. It's hidden from the (presumably incomplete) theory that we currently have.

In the case of a single bit, or single particle spin, or whatever – there's really not much to argue one way or the other.

Entanglement Correlations

Things get more interesting when we consider entangled systems. Take a pair of particles in the following spin state along the z axis:

$$\frac{1}{\sqrt{2}}|+z,+z\rangle + \frac{1}{\sqrt{2}}|-z,-z\rangle$$

Suppose the first particle belongs to Alice and the second one to Bob. Then when they independently do z spin measurements, we know two things:

(1) They both have a 50/50 chance of getting spin up or spin down.
(2) They will both get the same results.

It's easy to see that these facts could be explained by spin values which are determined in advance of the measurements. When the two particles are originally entangled, you simply flip a coin. Heads you make them both spin up and tails you make them both spin down. There is an indeterminism (with specified probabilities) for the process of entanglement. But there is no further mystery about how results (1) and (2) are obtained.

However, when more complicated entangled states and measurements are done, it will turn out that no predetermined values will achieve the results prediced by quantum theory. This will be the subject of the next session, where we will look at something called *GHZ measurements.*

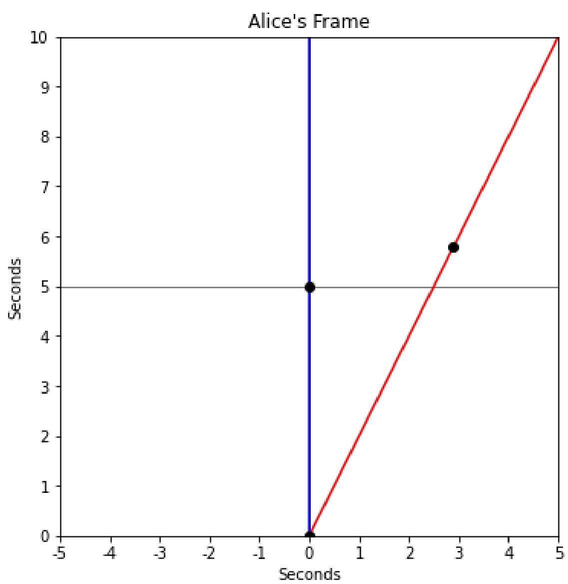
A sidebar on Special Relativity

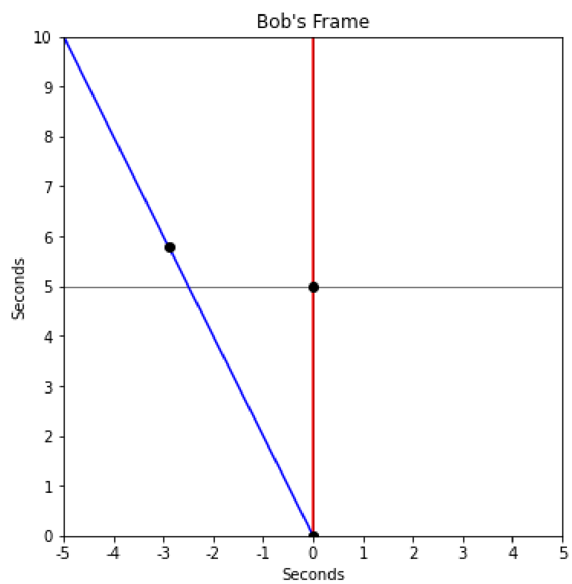
As a precursor to looking at more complicated states, let's take a brief side trip into relativistic physics, just to point out that the standard reading of special relativity *appears* to rule out the possibility that the particles are somehow communicating about their measurements.

Suppose Alice and Bob start out from the same point and travel apart from one another at half the speed of light, each carrying one of a pair of entangled particles. Then suppose that each one of them (from their own perspective) does a measurement on that particle 5 seconds after they pass each other.

Here are two spacetime diagrams, one from Alice's perspective and one from Bob's. As you can see (either from the spacetime diagrams or the Lorentz transformation below), both Alice and Bob see themselves as doing the measurement first. Due to the *relativity of simultaneity,* there is simply no fact of the matter about who went first.

This means that, according to relativistic physics as it is typically construed, statements such as "Alice's measurement changed the state of Bob's particle" are simply incoherent.





The Lorentz Transformation

Noting that we use the same units for time and space so that the speed of light equals one, the transformation is: $x' = \gamma(x - vt)$, $t' = \gamma(t - vx)$, where v is the relative velocity between frames and $\gamma = 1/\sqrt{1 - v^2}$. Alice and Bob each do a measurement at $x = 0, t = 5$ in their own frame of reference. They are moving at half the speed of light relative to one another, Alice to the left and Bob to the right. So Alice sees Bob's measurement at approximately $x = 2.89, t = 5.77$ and Bob sees Alice's at about $x = -2.89, t = 5.77$.

Final terminological note

Although here I used the term "Hidden Variables" to describe *preset values* for things like the state of a qubit or the spin of a particle, the expression is typically used to describe *any* approach which adds something to quantum theory in order to provide a more complete description of reality. So, as the term is commonly used, someone who believes in hidden variables simply believes that quantum theory is *incomplete* in one way or another, and that it can possibly be completed. For example, if it turned out that the values of Alice and Bob's measurements above *were* somehow being communicated (despite what relativity appears to say), that would also be considered a hidden variable theory.