The Hardy State

May 16, 2023

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The Hardy State is another state that shows predetermined variables can't reproduce the predictions of quantum theory, but in a different way than GHZ. It's not that you can't find a predetermined setting that will work, but rather that you *can get a result* that no "legal" predetermined setting could provide.

This short note assumes that you've been following the 2023 lecture series, that you understand the GHZ experiment, and know how to do all the basis changes to produce the states below. Any questions about all that, just email me at the address at the top.

Details

$$\text{hardy}_{zz} = \frac{1}{\sqrt{3}}|+z,+z\rangle + \frac{1}{\sqrt{3}}|+z,-z\rangle + \frac{1}{\sqrt{3}}|-z,+z\rangle$$

$$\text{hardy}_{zx} = \frac{\sqrt{2}}{\sqrt{3}}|+z,+x\rangle + \frac{1}{\sqrt{6}}|-z,+x\rangle + \frac{1}{\sqrt{6}}|-z,-x\rangle$$

$$\text{hardy}_{xz} = \frac{\sqrt{2}}{\sqrt{3}}|+x,+z\rangle + \frac{1}{\sqrt{6}}|+x,-z\rangle + \frac{1}{\sqrt{6}}|-x,-z\rangle$$

$$\text{hardy}_{xx} = \frac{\sqrt{3}}{2}|+x,+x\rangle + \frac{\sqrt{3}}{6}|+x,-x\rangle + \frac{\sqrt{3}}{6}|-x,+x\rangle - \frac{\sqrt{3}}{6}|-x,-x\rangle$$

Unlike the GHZ experiment, it's easy for Alice and Bob to choose predetermined values which will work for a single experiment. For example, they can just set all their variables to plus.

It's simple to show, however, that no predetermined values can reproduce the results of repeated experiments. Just consider the $|xx\rangle$ state. If both Alice and Bob do repeated X spin measurements, they will both see spin down (minus) results one out of twelve times.

But, when you consider the other three states, there is no way that Alice and Bob can both set their X variables to minus:

```
az ax bz bx
-- -- -- --
+  -  +  -  +z,-x ruled out by |zx⟩
+  -  -  -  +z,-x ruled out by |zx⟩
-  -  +  -  -x,+z ruled out by |xz⟩
-  -  -  -  -z,-z ruled out by |zz⟩
```

Notebooks

hardy-state.ipynb

References

Mermin QC book, Appendix D (p.175 in my copy)

Nonlocality for Two Particles without Inequalities for Almost All Entangled States
Lucien Hardy
PHYSICAL REVIEW LETTERS — VOLUME 71, NUMBER 11 — 13 SEPTEMBER 199
(I find the original paper hard to understand.)