Two-Dimensional Traveling Waves - Examples

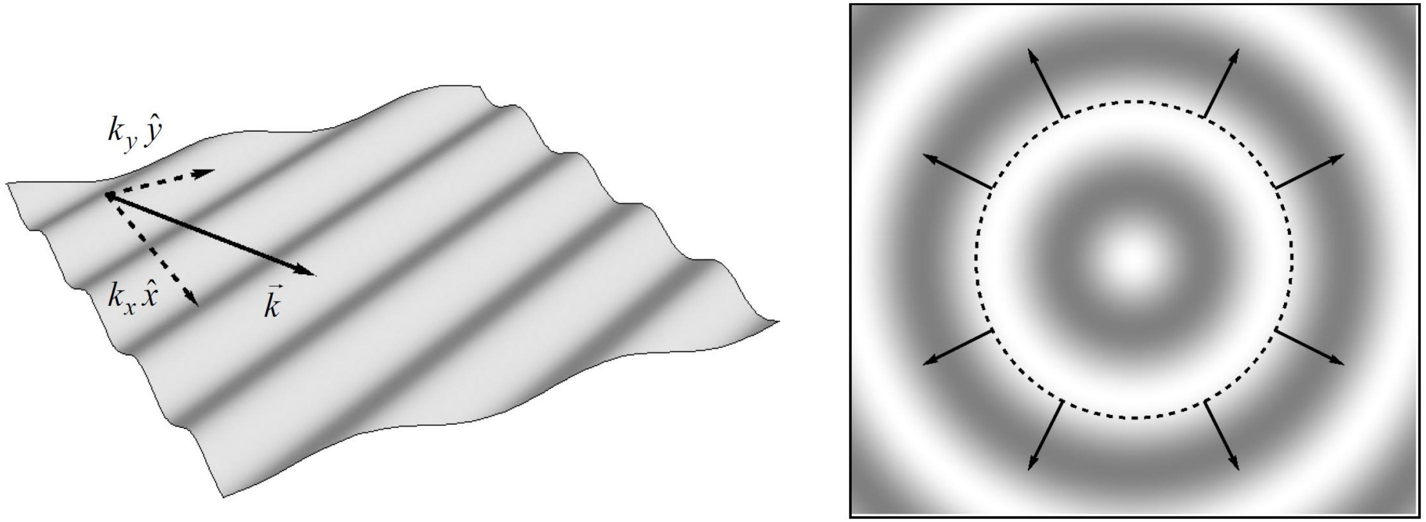


Figure D-1: (Left) a transverse plane wave propagating along a surface in the direction given by its wave vector $\vec{k}$. Note that the magnitudes of the wave vector's components give the rate of change of the wave's phase along their respective directions. (Right) a circular wave propagating outward. A selection of wave vectors is shown; each vector is perpendicular to the surface of constant phase at its location (in this case, a circle centered on the source).

Two Dimensional Standing Waves - Examples

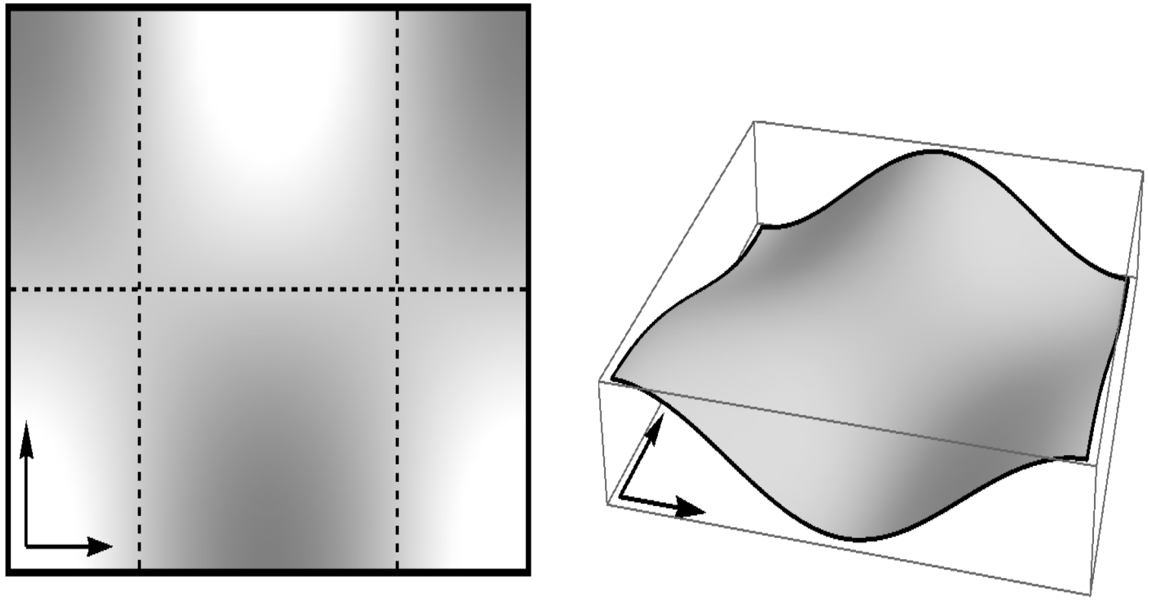


Figure D-2: Two views of a standing wave mode in a rectangular resonant cavity with "free" (Neumann) boundary conditions. The nodal lines of the mode are shown (dashed) in the left graphic; the arrows in each graphic represent $\hat{x}$ and $\hat{y}$ unit vectors. In this example such solutions may be constructed by adding a symmetric set of plane waves all with the same wave number $|\vec{k}|$ as described in the text and shown in Figure D-3.

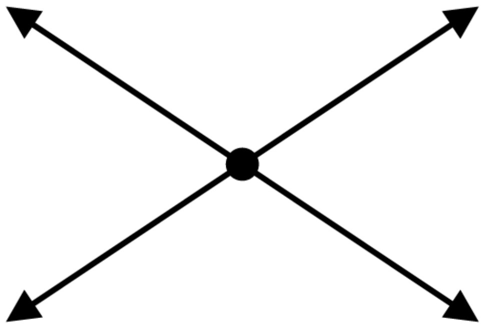


Figure D-3: Wave vectors of the four traveling waves whose sum provides the standing wave solution shown in Figure D-2.

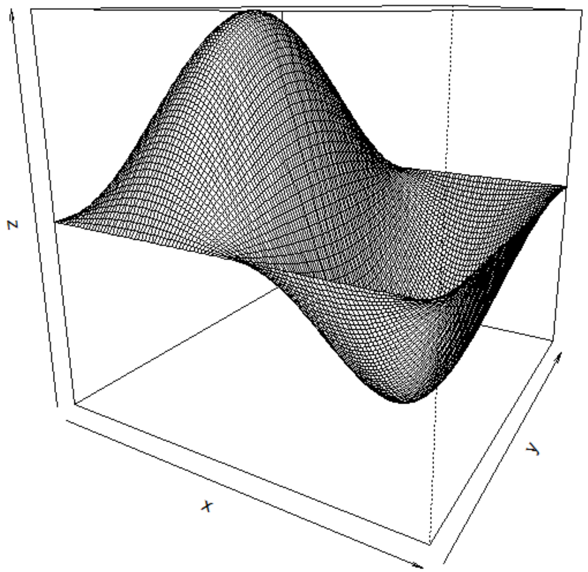
EM Waves For Blackbody Box

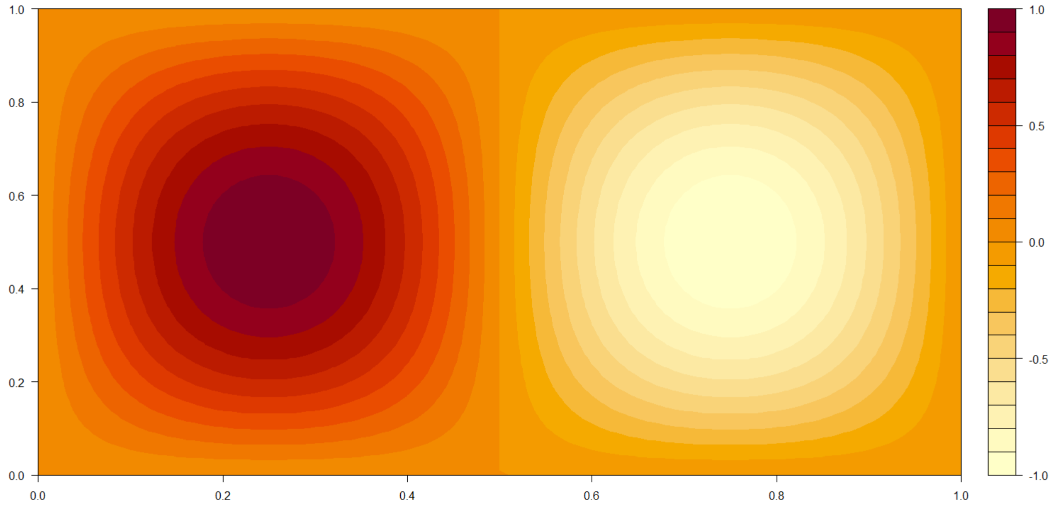
- Standing EM wave solution to the wave equation for a cubic box.
- $\nabla^2\psi = \alpha^2\psi/\alpha x^2 + \alpha^2\psi/\alpha y^2 + \alpha^2\psi/\alpha z^2 = \frac{1}{c^2} * \alpha^2\psi/\alpha t^2$
- $\psi = \sum_{\vec{k}} \begin{bmatrix} \sin(k_x x) * \sin(k_y y) * \sin(k_z z) * \\ \left(A_{\vec{k}} * \cos(w_{\vec{k}} t) + B_{\vec{k}} * \sin(w_{\vec{k}} t)\right) \end{bmatrix}$
- Index of $\vec{k}$ is the set $\{k_x, k_y, k_z\}$
- $k_d = \frac{\pi n_d}{L}$ - wave must go to zero at the box boundaries in each Cartesian direction.

EM Waves For Blackbody Box

- $w_{\vec{k}} = 2\pi f_{\vec{k}} = \ 2\pi c/\lambda_{\vec{k}} = 2\pi c/\left(2\pi/\left|\vec{k}\right|\right) = c\left|\vec{k}\right|$
- $\left|\vec{k}\right| = \ \sqrt{k_x^2 + \ k_y^2 \ + \ k_z^2} \ = \ \frac{2\pi}{\lambda}$ - magnitude of the wave number – Pythagorean sum of the Cartesian components.

Example of Standing Wave in a Square

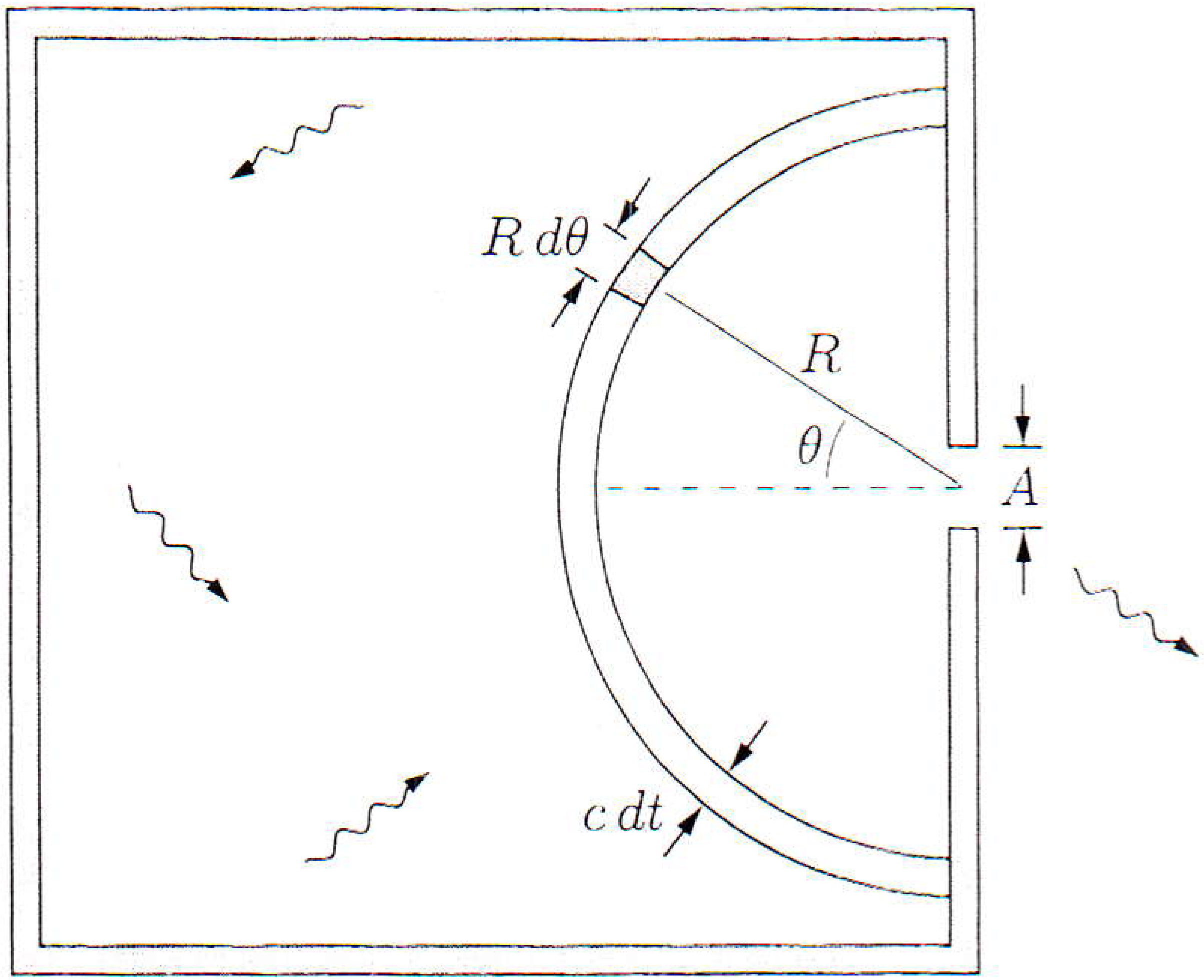




Two dimensional version, with wave displacement in the z direction. Ignore time. Box slide length set to one unit. Amplitude set to one unit.

Select solution of n = 2 in the x direction, and n = 1 in the y direction.

Energy Escape



Energy Escape

- All escaping energy originating at distance R reaches the cavity at the same time (R/c) – hemispheric shell.
- Energy in chunk = (energy density) * (volume of chunk).
- $= \frac{E}{V} * \left(R^2 \sin(\theta)\, d\theta d\varphi\right) * (c\, dt)$
- Fraction of energy escaping = ½ * (effective cross sectional area of the cavity) * 1/(surface area of the hemisphere).
- $= \frac{1}{2} * A\cos(\theta) * 1/\left(2\pi R^2\right)$

Energy Escape

- Factor out A and dt, as we are want the intensity = escaping energy per unit time per unit area.
- Integrate over the surface of the hemisphere.
- $I = \frac{1}{A} * \ d(energy\ escape)/dt$
- $= \ \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \left\{ \left[\frac{E}{V} \ * R^2 \sin(\theta) * c \right] \ * \ \left[\frac{1}{2} \ * \cos(\theta) * \ 1/\left(2\pi R^2\right) \right] \right\} d\varphi d\theta$
- $= \frac{E}{V} \ * c/(4\pi) * \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \sin(\theta) \cos(\theta) \, d\varphi d\theta$
- $= \frac{E}{V} \ * c/(4\pi) * 2\pi * \ \int_0^{\frac{\pi}{2}} \sin(\theta) \cos(\theta) \, d\theta$

Energy Escape

- $= \frac{E}{V} * \frac{c}{2} * \int_0^{\frac{\pi}{2}} \sin(\theta)\cos(\theta)\, d\theta = \frac{E}{V} * \frac{c}{2} * \frac{1}{2}\, sin^2(\theta)|_0^{\frac{\pi}{2}} =$
- $\frac{E}{V} * \frac{c}{2} * \frac{1}{2} = \frac{c}{4} * \frac{E}{V} = I$