Time Dilation and Length Contraction Answers

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Muons

A muon is a particle typically created in the upper atmosphere and has a half-life of about 1.5 μs. (This means that if you take a bunch of muons and wait 1.5 μs, about half of them will have decayed and about half will still be around.) A muon travels at .99 the speed of light.

So, given that a muon has been created 2,000 meters above the Earth, it would seem to have a very small chance of reaching the ground and being detected.

Say that such a muon does reach the Earth and is detected in a laboratory. In the Lab reference frame, the muon has travelled 2,000 m. So based on its velocity, it has apparently lived for about 6.734 μs in the Lab frame of reference.

Problem 1: How long has this muon lived in its own frame of reference?

$\gamma = \dfrac{1}{\sqrt{1 - .99^2}} \approx 7.089$

The muon is present at its creation event and when it arrives at the lab. So the muon's time between these events is the proper time, which will be the shortest time. This means we need to divide the lab time be γ to get the muon's time.

$6.734/7.089 \approx$ **.95 microseconds.**

Problem 2: How far has it traveled in its own frame of reference?

The proper length is the distance in the Lab frame, so we just contract that to get the distance the muon sees.

$2,000/7.089 \approx$ **282 meters.**

The Qzzyyxxx

Scientists on Earth detect an asteroid-sized object hurtling toward us at 80% of the speed of light. Fortunately the path of the object (an inertial one by the way) takes it a ways outside the orbit of the moon, and it actually doesn't end up presenting any danger.

But as the object passes by, it beams a message saying "Greetings, people of Earth. We, the Qzzyyxxx empire, salute you and wish you well as we pass you by on our way to visit your close neighbors in the Proxima Centauri system 4 light years away."

The astounded scientists continue to track the Qzzyyxxx ship (using the new Chinese "Xiezhi" super-array telescope) until it reaches the Proxima Centauri system and disappears from view.

Problem 3: How long does it take the Qzzyyxxx to travel the 4 light years from Earth to Proxima Centauri in the Earth's frame of reference?

They're travelling at .8 the speed of light, so it takes them $4/.8 =$ **5 years**.

Problem 4: How long does it take them in the Qzzyyxxx ship's frame?

$$\gamma = \frac{1}{\sqrt{1 - .8^2}} \approx 1.667$$

The Qzzyyxx are present at both the start and end of the journey between Earth and Proxima Centauri, so they will see the proper (shortest) time between those events. Therefore we divide the Earth time by γ, giving: $5/1.667 \approx$ **3 years**

Problem 5: How far is the journey from Earth to Proxima Centauri in the Qzzyyxxx frame?

We'll assume that the Earth and Proxima Centauri are not moving enough with respect to each other to make a difference. Then the distance seen by Earth is the proper (greatest) one. We divide it by γ giving: $4/1.667 \approx$ **2.4 light years.**