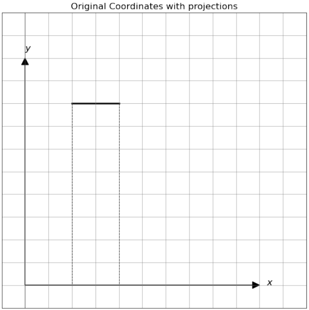
Brief (Partial) Review of Special Relativity

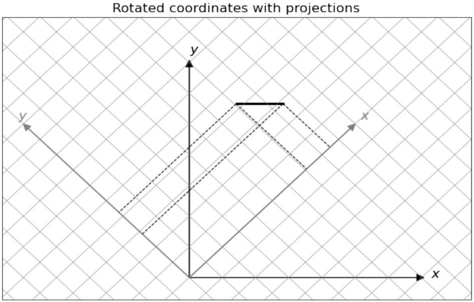
September 5, 2025

The Spacetime Perspective

- We are going to take the "Space-time perspective."
- There are other perspectives, but we'll start with this one.
- Space and time together form a four dimensional space.
- But this space "works differently" than Euclidian space.
- The spacetime of Special relativity is sometimes called "Minkowski space."
- Rotations in Minkowski space work differently than in Euclidian space.

Rotations in the X/Y plane





The (proper) length of the bar doesn't change, but when you view it from a different coordinate system it appears "contracted."

Units and Conversions

We think of time being measured in seconds and space being measured in meters. But if space and time are joined together then it makes more sense to use the same units for both.

- The Taylor "Dam example."
- $C \approx 3 \times 10^8$ meters/second is a conversion factor.
- seconds * C = meters
- meters / C = seconds
- Why C? The "speed" of light.
- Additional material if needed: Units.ipynb.

- Reference frames. The standard setup is that the primed frame moves "to the right" with velocity v relative to the unprimed frame.

- the Galilean transformation:

  $x' = x - vt$

  $t' = t$

- The Lorentz transformation:

  $x' = \gamma(x - vt)$

  $t' = \gamma(t - vx)$

  where $\gamma = \dfrac{1}{\sqrt{1 - v^2}}$

  Notice that eliminating C makes the natural symmetry of the Lorentz transform apparent.

- The Lorentz transform represents a **rotation between space and time.** But spacetime is not Euclidean, and these "rotations" are different than rotations in regular space.

A Moving Rod

- Both space and time are in units of meters. $\gamma = \frac{1}{\sqrt{1-v^2}}$.
- I am in an inertial frame (the unprimed frame).
- There is a moving rod (the primed frame).
- The rod had velocity v relative to me, in the $+x$ direction.
- The rod is L meters long in its own frame.
- The head of the rod passes me at $x = 0, t = 0$ in both frames.
- At $t = 0$ in the primed frame the head is at $x = 0$ and the tail is at $x = -L$.
- I do the Lorentz transform on the tail of the rod:

  $x = \gamma(x' + vt') = \gamma(-L + 0) = -\gamma L$

  $t = \gamma(t' + vx') = \gamma(0 - vL) = -\gamma vL$
- So the tail of the rod is at $x = -\gamma L$ at $t = -\gamma vL$ in my frame of reference.
- Between then and my t=0, the tail travels $vt = v^2\gamma L$ in my frame of reference.
- Giving the tail postion at my $t = 0$ as: $-\gamma L + v^2\gamma L = (v^2 - 1)\gamma L = -(1 - v^2)\gamma L$.
- This works out to $-L/\gamma$, giving a rod length of L/γ in my frame.