Units and Conversions

Contents

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^ Setup

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Autosave disabled
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This next cell hides all the input cells.
To get them back: Kernel -> Restart & Clear Output

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^ Time and Space

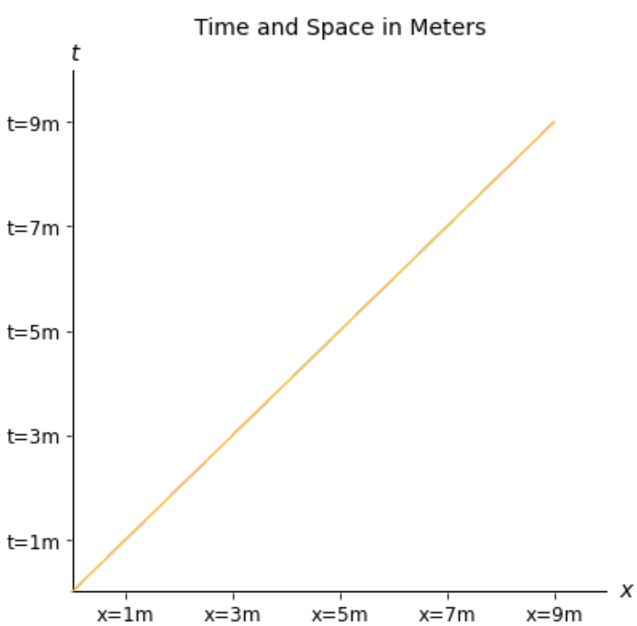
The basic Lorentz tranforms are:

$$\gamma = \frac{1}{\sqrt{1 - v^2}}$$

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - vx)$$

These work (for the standard configuration on the x axis) if we have time and space in the same units. For example: meters and meters, or seconds and "light seconds."



If we have distance in meters and time in seconds, then we need to do some conversions before we depict them on a spacetime diagram.

Since we are taking it as a basic assumption that light "travels" the same distance in time and space, then we can use that to convert between seconds and meters.

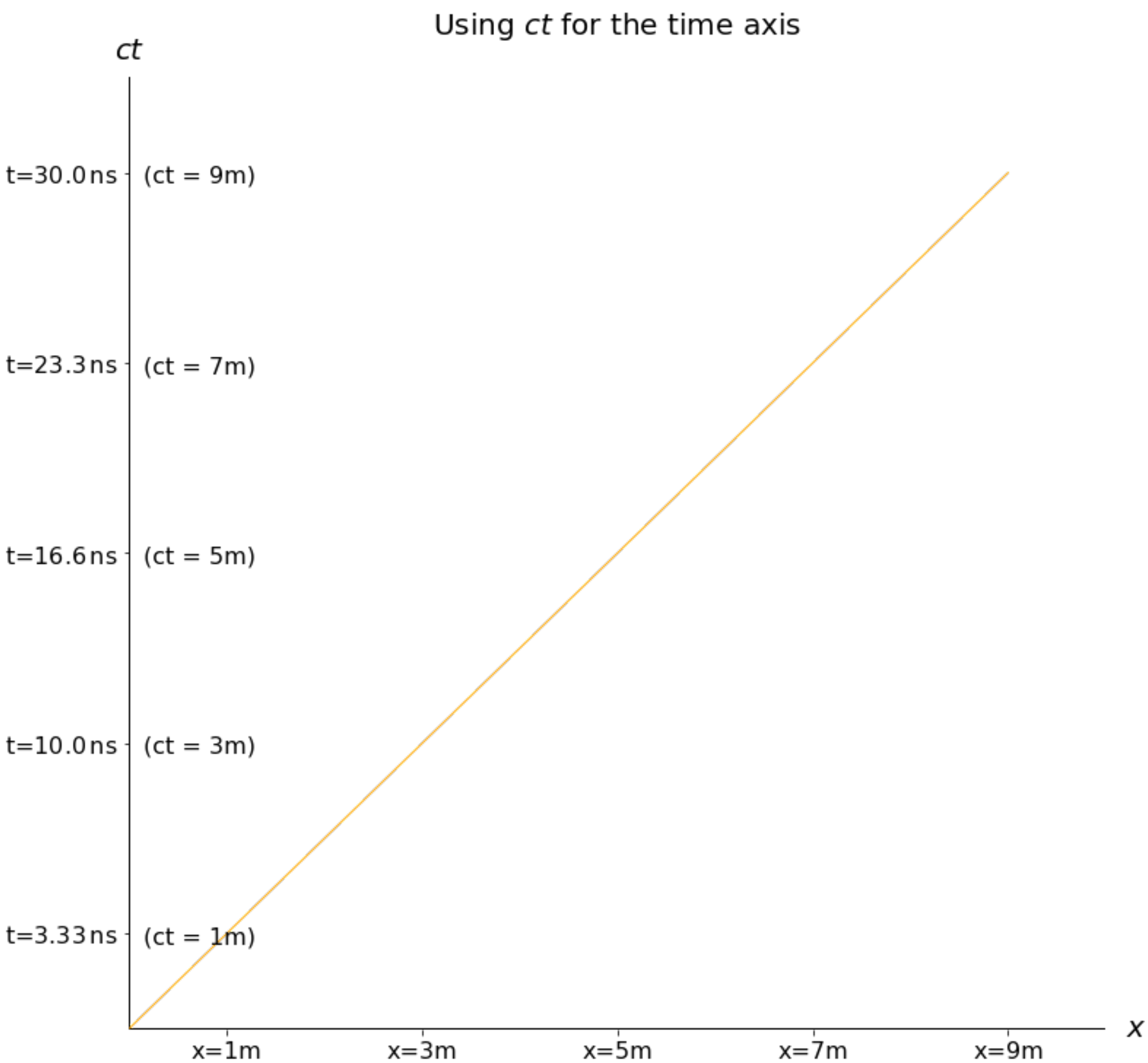
> Multiply 1 second times C
>
> $1s \times 3 \times 10^8 \frac{m}{s}$
>
> The seconds cancel, so we have ...
>
> 1 second = 3×10^8 meters
>
> And therefore ...
>
> 1 meter = 1 second / 3×10^8
>
> 1 meter = 3.33×10^{-9} seconds (just over 3 nanoseconds)

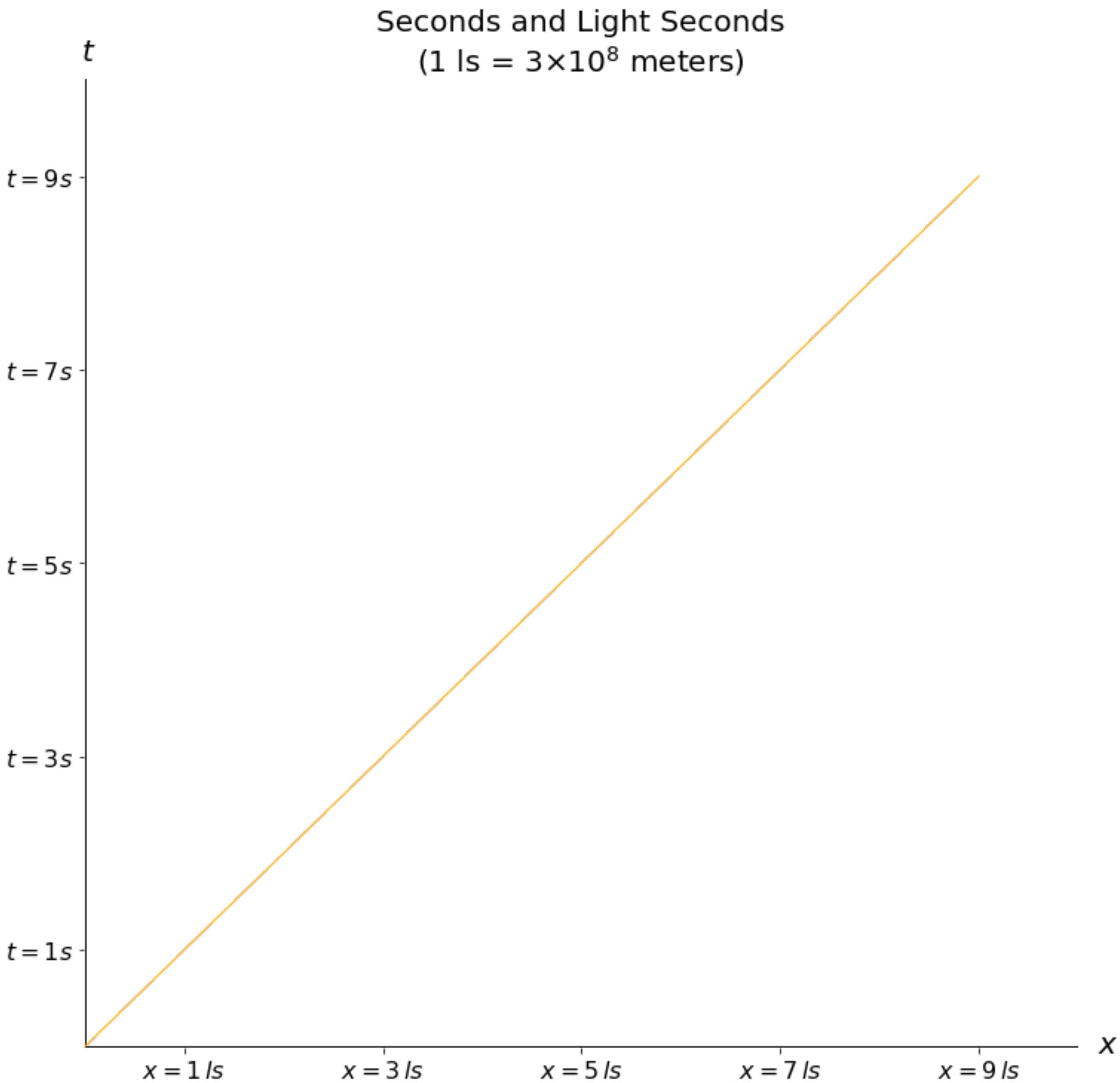
But the key point is that:

> Seconds * C = Meters
>
> Meters / C = Seconds

So to take the Lorentz transformations and adjust them for x given in meters and t given in seconds, we need to convert the t values to meters by multiplying them by C. Bearing in mind that $v = x/t$, we do the replacements:

> We start with:
>
> $\gamma = \frac{1}{\sqrt{1 - v^2}}$
>
> $x' = \gamma(x - vt)$
>
> $t' = \gamma(t - vx)$
>
> Replacing t by ct gives ...
>
> $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
>
> $x' = \gamma(x - \frac{v}{c}ct)$
>
> $ct' = \gamma(ct - \frac{v}{c}x)$
>
> Canceling c in x' and dividing through by c in t' gives ...
>
> $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
>
> $x' = \gamma(x - vt)$
>
> $t' = \gamma(t - \frac{vx}{c^2})$


Using *ct* for the time axis
ct
t=30.0 ns (ct = 9m)
t=23.3 ns (ct = 7m)
t=16.6 ns (ct = 5m)
t=10.0 ns (ct = 3m)
t=3.33 ns (ct = 1m)
x=1m
x=3m
x=5m
x=7m
x=9m
x


Seconds and Light Seconds
(1 ls = 3×10^8 meters)
t
t = 9s
t = 7s
t = 5s
t = 3s
t = 1s
x = 1 ls
x = 3 ls
x = 5 ls
x = 7 ls
x = 9 ls
x